

# Final State Interactions and $\varepsilon'/\varepsilon$ : A Critical Look

A.J. Buras<sup>1</sup>, M. Ciuchini<sup>1,2</sup>, E. Franco<sup>1,3</sup>,  
G. Isidori<sup>1,4</sup>, G. Martinelli<sup>3,5</sup> and L. Silvestrini<sup>1,3</sup>

<sup>1</sup> Physik Department, Technische Universität München, D-85748 Garching, Germany

<sup>2</sup> Dipartimento di Fisica, Università di Roma Tre and INFN, Sezione di Roma Tre,  
Via della Vasca Navale 84, I-00146 Rome, Italy

<sup>3</sup> Dipartimento di Fisica, Università di Roma “La Sapienza” and INFN, Sezione di Roma,  
P.le A. Moro 2, I-00185 Rome, Italy

<sup>4</sup> INFN, Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Italy

<sup>5</sup> LAL and Laboratoire de Physique Théorique, Laboratoire associé CNRS,  
Université de Paris XI, Bât 211, 91405 Orsay Cedex, France

## Abstract

We critically analyze recent attempts to include final state interaction (FSI) effects in the calculation of the CP violating ratio  $\varepsilon'/\varepsilon$ . In particular the approach of using the Muskhelishvili-Omnès equation for the decay amplitudes is examined. We find that the values of the dispersive correction factors are very sensitive to initial conditions which are, at present, out of control, and demonstrate that the claimed large enhancement of  $\varepsilon'/\varepsilon$  through FSI is questionable. We propose, instead, a different implementation of this approach which may be useful to lattice determinations of weak amplitudes.

# 1 Introduction

One of the crucial issues for our understanding of CP violation is the question whether the size of the observed direct CP violation in  $K_L \rightarrow \pi\pi$  decays, expressed in terms of the ratio  $\varepsilon'/\varepsilon$ , can be described within the Standard Model. Experimentally the grand average, including the results from NA31 [1], E731 [2], KTeV [3] and NA48 [4] reads

$$\text{Re}(\varepsilon'/\varepsilon) = (21.2 \pm 4.6) \times 10^{-4}. \quad (1)$$

There are different opinions whether this result can be accommodated within the Standard Model. In [5, 6] the central value of the predicted  $\varepsilon'/\varepsilon$  is typically by a factor of 2-3 below the data, with an estimated theoretical uncertainty of the order of  $60 \div 100$  %. The result in Eq. (1), then, can only be accommodated if all relevant parameters are chosen simultaneously close to their extreme values. Higher values of  $\varepsilon'/\varepsilon$ , compatible with the data within one standard deviation have been found, instead, in [7, 8]. Recent reviews can be found in [9].

More recently, following the previous work of Truong [10], it has been pointed out by Pallante and Pich [11] that the inclusion of final state interactions substantially affects the estimates of  $\varepsilon'/\varepsilon$ . The size of the effect is parametrized in terms of dispersive correction factors  $\mathcal{R}_I$  multiplying the corresponding isospin amplitudes  $A_I$ . With  $\mathcal{R}_0 \approx 1.4$  and  $\mathcal{R}_2 \approx 0.9$ , as found in their numerical evaluation, a substantial enhancement of  $\varepsilon'/\varepsilon$  was obtained. Indeed, by including these factors, the authors of Ref. [11] find that the central value  $\varepsilon'/\varepsilon = 7.0 \times 10^{-4}$  of [5] gets increased to  $\sim 15 \times 10^{-4}$  which is much closer to the experimental average (1). Similar results, although with a different dispersive approach, have been found in [12].

The interesting paper of Pallante and Pich stimulated us to give a closer look to the whole procedure followed to obtain the correction factors  $\mathcal{R}_I$ . The phenomenological application of the Muskhelishvili-Omnès equation to the  $K \rightarrow \pi\pi$  case requires some arguable assumptions (see Section 4). However, even within those assumptions, we found a major objection which unfortunately makes the calculation of Ref. [11] questionable. On the basis of symmetry arguments only, one can show that there is a point, out of the physical cut, where the amplitudes induced by left-handed operators are expected to vanish. As demonstrated below, the authors of Ref. [11] made an assumption on the value of the derivative of the amplitude at this point which is not justified. All their results rely on this assumption. We will show that, by replacing the latter condition with others, equally acceptable on physical grounds, it is possible to obtain very different values of the correction factors  $\mathcal{R}_I$ . Thus we conclude that the results found in [11] are subject to substantial uncertainties.

In spite of this, we found some interesting application of the dispersive analysis to non-perturbative calculations of weak amplitudes. For example, lattice calculations could provide a condition which replaces the arbitrary assumption on the derivative of the amplitude and fix unambiguously the factors  $\mathcal{R}_I$  (under the hypotheses inherent to the use of the Muskhelishvili-Omnès equation in  $K \rightarrow \pi\pi$  decays).

Our paper is organized as follows. In section 2 we present general formulae for the Muskhelishvili-Omnès solution of dispersion relations applied to  $K \rightarrow \pi\pi$  amplitudes, focusing on the assumptions needed to obtain the final result. Using these formulae we demonstrate in Section 3 that the evaluation of the dispersive factors  $\mathcal{R}_I$  along the lines proposed in [11] is ambiguous. In particular we stress that there is no justification to use the numerical values of  $\mathcal{R}_I$  found in [11] in conjunction with present lattice and large- $N_C$  calculations of hadronic matrix elements. Similar comments apply to [12]. In Section 4 we briefly illustrate how modifications of the technique developed in [10, 11] could in principle improve the accuracy of the predictions for  $\varepsilon'/\varepsilon$  in conjunction with future lattice calculations. We conclude in Section 5.

## 2 General formulae

In this section we recall the basic ingredients used to derive the solution of the Muskhelishvili-Omnès equation [13] in the case of  $K \rightarrow \pi\pi$  amplitudes. The starting point is the  $N$ -subtracted dispersion relation. We denote by  $A(s)$  the decay amplitude of a kaon of mass  $m_K = \sqrt{s}$  into two pions of mass  $m_\pi$ . The invariant mass of the two pions is taken as  $(p_1 + p_2)^2 = s$ , corresponding to the insertion of a weak operator carrying zero momentum. The following discussion applies as well to the case where  $A(s)$  is the  $K \rightarrow \pi\pi$  matrix element of a local operator renormalized at a fixed scale  $\mu$ .

Following Ref. [10] we assume that  $A(s)$  is analytic in the cut  $s$  plane, with the cut going from  $4m_\pi^2$  to  $\infty$ <sup>1</sup>. Further assuming that  $s^{-N}A(s) \rightarrow 0$  for  $s \rightarrow \infty$ , we can write

$$A(s) = P_{N-1}(s, s_0) + \frac{(s - s_0)^N}{\pi} \int_{4m_\pi^2}^{\infty} dz \frac{\text{Im}A(z)}{(z - s_0)^N (z - s - i\epsilon)}, \quad (2)$$

where  $P_{N-1}(s, s_0)$  is a  $N - 1$  degree  $s$  polynomial, the coefficients of which are to be fixed by  $N$  independent conditions on the amplitude and  $s_0$  denotes the subtraction point (in general one could choose  $N$  different subtraction points), which has to lie outside the physical cut. Writing the imaginary part of the amplitude as

$$\text{Im}A(s) = A(s)e^{-i\delta(s)} \sin \delta(s), \quad (3)$$

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<sup>1</sup> These analyticity properties of  $A(s)$  are certainly not fully correct. However one can argue that Eq. (2) still holds to a reasonable accuracy in a region close to the physical point (see Section 4).

the dispersion relation (2) becomes a Muskhelishvili-Omnès equation [13]. Here  $\delta(s)$  is the strong phase of the amplitude, which in the elastic region is equal to the phase shift of the  $\pi\pi$  scattering appropriate to a given isospin channel <sup>2</sup>. The general solution of this equation is given by [13]

$$A(s) = Q_{N-1}(s) O(s), \quad (4)$$

where

$$O(s) = \exp \left( \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dz \frac{\delta(z)}{(z - s - i\epsilon)} \right) \quad (5)$$

and  $Q_{N-1}(s)$  is a  $N-1$  degree polynomial. Equation (4) shows explicitly that  $N$  independent conditions on  $A(s)$  and the knowledge of the phase on the whole cut are sufficient to fully determine the amplitude. Note that there is no longer any reference to the subtraction point  $s_0$  of the original dispersion relation.

We point out that, if we were able to obtain a number of conditions on  $A(s)$  larger than  $N$ , we could reduce the sensitivity of the solution to the knowledge of  $\delta(s)$  at large  $s$ . Indeed, we can always rewrite the solution (4) as

$$\begin{aligned} A(s) &= Q_{N-1}(s) \exp \left( \sum_{i=0}^M c_i s^i \right) \\ &\times \exp \left( \frac{s^{M+1}}{\pi} \int_{4m_\pi^2}^{\infty} dz \frac{\delta(z)}{z^{M+1}(z - s - i\epsilon)} \right), \end{aligned} \quad (6)$$

where the product of the first two terms is completely determined by  $N+M$  independent conditions. If possible, this could be useful since, in practice,  $\delta(s)$  can be extracted from the data only up to the inelastic threshold.

To make contact with the work of Pallante and Pich [11], we now discuss two examples on how  $A(s)$  is determined from a specific set of conditions in the case  $N=2$ . In this case, corresponding to an amplitude going at most linearly in  $s$  for large  $s$ , two conditions on the amplitude are required:

1. In order to fix the coefficients of  $Q_1(s)$ , a possible set of conditions is given by the knowledge of the amplitudes at two different points  $s_1$  and  $s_2$ , namely

$$A(s_1) = A_1, \quad A(s_2) = A_2. \quad (7)$$

In this case, one finds

$$A(s) = \frac{A_1 O(s_2) (s - s_2) + A_2 O(s_1) (s_1 - s)}{(s_1 - s_2) O(s_1) O(s_2)} O(s). \quad (8)$$

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<sup>2</sup> Note that even in the elastic region  $\delta(s)$  can be identified with the phase shift measured from  $\pi\pi$  scattering only if the dependence of the latter on the kaon mass is negligible. We thank G. Colangelo for pointing this out to us.

We stress that, although the r.h.s of this equation is written in terms of  $s_{1,2}$ , the physical amplitude is obviously independent of the choice of these points, provided that  $A_{1,2}$  are given by Eq. (7).

2. A different set of conditions can be given by

$$A(s_1) = 0, \quad A'(s_1) = C, \quad (9)$$

namely by knowing the point  $s_1$  where the amplitude vanishes, or assumes a specific value, and its first derivative in that point. With the conditions in Eq. (9) the solution is particularly simple and takes the form

$$A(s) = C(s - s_1) O(s_1)^{-1} O(s). \quad (10)$$

### 3 Why we are not able to compute universal dispersive factors

We now critically examine the procedure followed by Pallante and Pich [11] to compute the so-called *dispersive factors*  $\mathcal{R}_I$  for the  $I = 0$  and  $I = 2$  amplitudes. To this end, we start by summarizing the main steps of the procedure adopted in Ref. [11].

On the basis of the lowest-order chiral Lagrangian, Pallante and Pich assume that the solution of the Muskhelishvili-Omnès equation is given by

$$A_I(s) = C_I(s - m_\pi^2) \Omega_I(s), \quad (11)$$

where

$$\Omega_I(s) = \frac{O_I(s)}{O_I(m_\pi^2)} \equiv e^{i\delta_I(s)} \mathcal{R}_I(s). \quad (12)$$

Here  $O_I$  is given by Eq. (5) with  $\delta$  replaced by  $\delta_I$ , the experimental elastic  $\pi\pi(I)$  phase shift, integrated up to a certain cutoff. This solution corresponds to the case in Eq. (10) with  $s_1 = m_\pi^2$ , i.e. the authors of Ref. [11] impose  $A_I(m_\pi^2) = 0$  and implicitly assume the knowledge of  $A'_I(m_\pi^2) = C_I$ .<sup>3</sup> In order to fulfill the latter condition, they assume without justification that the matrix element at the physical point,

$$\mathcal{M}_I = \langle \pi\pi(I) | \mathcal{H}_{\text{eff}} | K \rangle|_{s=m_K^2}, \quad (13)$$

provided by non-perturbative methods such as lattice QCD or large  $N_C$  estimates, corresponds to  $C_I(s - m_\pi^2)|_{s=m_K^2}$ . Their solution can thus be written as

$$A_I(s) = \mathcal{M}_I \frac{(s - m_\pi^2)}{(m_K^2 - m_\pi^2)} e^{i\delta_I(s)} \mathcal{R}_I(s) \quad (14)$$

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<sup>3</sup> The need of these two assumptions is common also to Ref. [10], as pointed out in [14].

and the dispersive factors  $\mathcal{R}_I$  are simply given by

$$\mathcal{R}_I \equiv \mathcal{R}_I(m_K^2) = \left| \frac{A_I(m_K^2)}{\mathcal{M}_I} \right| = \left| \frac{O_I(m_K^2)}{O_I(m_\pi^2)} \right|. \quad (15)$$

We have the following comments on this approach:

- One of the two conditions used to determine the solution is  $A_I(m_\pi^2) = 0$ . This is justified in Ref. [11] on the basis of the lowest-order chiral realization of  $(8_L, 1_R)$  and  $(27_L, 1_R)$  operators. The zero at  $s = m_\pi^2$  of the amplitudes induced by these operators actually holds beyond the lowest order [15, 16]. Indeed, it is a consequence of the vanishing of these amplitudes in the  $SU(3)$  limit, as follows by an old result by Cabibbo and Gell-Mann [17]. In modern language, we can rephrase the result of Ref. [17] by saying that the on-shell  $K \rightarrow 2\pi$  matrix element of any local  $(8_L, 1_R)$  operator invariant under  $CPS$  symmetry [18] vanishes in the absence of  $SU(3)$  breaking. Therefore we think that this condition is well justified in the case of  $(8_L, 1_R)$  operators and, particularly, in the case of  $Q_6$  considered in [11].
- Our main criticism is related to the determination of the second condition on  $A_I(s)$ , namely the one on  $A'_I(m_\pi^2)$ . Chiral perturbation theory alone cannot fix the value of this derivative, which must be known from some non-perturbative information. In their procedure, Pallante and Pich implicitly assume that the matrix elements determined by non-perturbative methods such as lattice or large  $N_C$ , fix the value of  $A'_I(m_\pi^2)$  according to the condition

$$A'_I(s = m_\pi^2) = C_I = \frac{\mathcal{M}_I}{m_K^2 - m_\pi^2}. \quad (16)$$

This assumption, which may look reasonable on the basis of the lowest-order chiral Lagrangian, actually involves an ambiguity of the same order of the dispersive correction itself. To illustrate this point, we make a different choice of the initial conditions (while keeping the same  $\mathcal{M}_I$ ): we use  $\mathcal{M}_I$  to fix the value of the amplitude at threshold, via the relation

$$A_I(s = 4m_\pi^2) = \mathcal{M}_I \frac{3m_\pi^2}{m_K^2 - m_\pi^2}. \quad (17)$$

In other words, we assume that  $\mathcal{M}_I(s - m_\pi^2)/(m_K^2 - m_\pi^2)$  provides a good approximation to the real amplitude near  $s = 4m_\pi^2$  instead of the point  $s = m_\pi^2$  as was implicitly employed in [11]. One may argue that this is also a reasonable choice because strong-interaction phases vanish at threshold, but of course this condition

is as arbitrary as the one adopted in [11]. Using this condition and  $A(s = m_\pi^2) = 0$ , from Eq. (8) we find

$$A_I(s) = \mathcal{M}_I \frac{(s - m_\pi^2) O_I(s)}{(m_K^2 - m_\pi^2) O_I(4m_\pi^2)}. \quad (18)$$

With the same parameterization of the phase  $\delta_0(s)$  as in Ref. [11] and our choice (18), the dispersive factor for  $(8_L, 1_R)$  operators is

$$\mathcal{R}_0^{\text{threshold}} = \left| \frac{O_0(m_K^2)}{O_0(4m_\pi^2)} \right| \simeq 1.1 \quad (19)$$

instead of

$$\mathcal{R}_0^{\text{PP}} = \left| \frac{O_0(m_K^2)}{O_0(m_\pi^2)} \right| \simeq 1.4. \quad (20)$$

With  $\mathcal{R}_0 = 1.1$  (and  $\mathcal{R}_2 = 1.0$ ) the central value  $\varepsilon'/\varepsilon = 7.0 \times 10^{-4}$  of [5] gets increased to  $\sim 9 \times 10^{-4}$ , instead of  $\sim 15 \times 10^{-4}$  found in [11]. While the first enhancement is well within the theoretical uncertainties quoted by the various analyses [5]–[8], the second one would have a considerable impact on the predicted value of  $\varepsilon'/\varepsilon$ .

Other choices of the initial conditions, equally acceptable under similar assumptions, would lead to still different results for  $\mathcal{R}_0$ . This exercise illustrates that, *unless one knows the value of  $s$  at which a given non-perturbative method provides the correct value of the amplitude (and/or its derivative)*, it is impossible to unambiguously compute the appropriate correction factor due to final state interactions.

- In the case of  $\Delta I = 3/2$  matrix elements of  $(8_L, 8_R)$  operators, such as  $\langle Q_8^{(3/2)} \rangle$  considered in Ref. [11], the amplitude does not vanish at  $s = m_\pi^2$ . Still the authors of [11] claim that the correction factor is  $\mathcal{R}_2 = |O_2(m_K^2)/O_2(m_\pi^2)| \simeq 0.9$ . We can reproduce this result assuming that the dispersion relation requires only one subtraction and that the non-perturbative calculations give the correct amplitude at  $s = m_\pi^2$ . This condition, however, appears even more arbitrary than the one used to determine  $A'(m_\pi^2)$  in the case of  $(8_L, 1_R)$  operators. Indeed in this case the point  $s = m_\pi^2$  does not play any special rôle. It is therefore not surprising that in a different but consistent framework, as the one discussed in [19, 20], non-perturbative corrections to  $\langle Q_8^{(3/2)} \rangle$  are found to enhance, rather than suppress, lattice and large- $N_C$  estimates.

Let us then summarize the main point we want to make. While the full amplitude clearly does not depend on the choice of the value of  $s$  at which the conditions on the amplitude are placed, the dispersive correction factors are obviously dependent on this

choice as we explicitly demonstrated above. As presently we are not in a position to state precisely which value of  $s$  the existing non-perturbative approaches, like lattice,  $1/N$ , etc., correspond to, it is not possible to determine uniquely the corresponding factors  $\mathcal{R}_I$  to be incorporated in each of those methods.

Before concluding this section, we comment on other attempts present in literature to include FSI effects in  $\varepsilon'/\varepsilon$ . The approach of Ref. [12] is somehow similar to the one of Ref. [11]. In this paper, however, the integral over the cut is divided in two parts: a low-energy region from threshold up to  $\mu^2$  and a high-energy region from  $\mu^2$  up to  $\infty$ . By identifying the scale  $\mu$  with the renormalization scale of the inserted operator, the high-energy region is claimed to be estimated in perturbation theory. The results thus obtained are in qualitative agreement with those of Ref. [11] and show a rough independence from the unphysical scale  $\mu$ . We stress, however, that the matching procedure adopted in this paper cannot be justified theoretically. Since we do not see a way of deriving consistently the formulae of Ref. [12] from first principles, we are skeptical about the approach followed in this paper.

Other recipes to include *a posteriori* FSI effects in non-perturbative calculations of weak amplitudes were presented in the recent literature [7, 21]. For these we have objections similar to those made to [11]: it is not clear to us how to relate the theoretical values of the amplitudes, computed with some non-perturbative method, to the physical ones which include FSI. In cases where the results of non-perturbative calculations are unable to reproduce the experimental phases, it is not possible to extract unambiguously the absolute value of the amplitudes. Since corrections may be applied only if this issue is under control, we think these procedures to incorporate empirically correcting factors to account for FSI do not help to improve the accuracy of the calculation. These attempts provide at best estimates of the theoretical uncertainties, as for instance discussed in [8].

## 4 Further developments

In spite of the criticism discussed before, we believe the Truong-Pallante-Pich approach is interesting and deserves further investigation. For this reason, in this section we briefly discuss possible applications of this technique to improve the accuracy of theoretical predictions for  $K \rightarrow \pi\pi$  matrix elements. We first want to state the assumptions needed to make this approach useful:

1. In general, in order to fix the number  $N$  of subtractions in the dispersion relation, hence the degree of the polynomial  $Q_{N-1}$ , one has to make an ansatz on the behaviour of the amplitude as  $s \rightarrow \infty$ . In the present case, however, Eq. (2) should



be understood as an approximate equation, valid in a limited  $s$  region close to the physical point. Indeed other sources of non-analiticity besides the cut starting at  $4m_\pi^2$  can appear in  $A(s)$  due to the identification of  $s$  with kaon mass, that is a parameter of the underlying theory. Still the solution of Eq. (2) can give a good estimate of the true amplitude if these further sources of non-analiticity are well represented by a polynomial expansion in the considered region. This is for instance the case in an effective theory where the pions are the only dynamical fields, or in the quenched approximation of QCD. In this context the number of subtractions is no longer related to the behaviour of the amplitude at large  $s$ , rather to the accuracy one needs to achieve. Large values of  $N$  are obviously preferable, although in practice they are hardly attainable. Therefore, following Refs. [10, 11], we assume that  $N = 2$  provides a reasonable approximation.

2. Above the inelastic threshold,  $\delta(s)$  is no longer the  $\pi\pi$  phase shift and cannot be directly extracted from the experimental data. In addition, even in the elastic region, the phase of  $A(s)$  can be identified with the experimental  $\pi\pi$  phase shift only assuming that the dependence of the  $\pi\pi$  scattering amplitude on the kaon mass is negligible. Were we able to compute the weak amplitude for a large number of points, we could reduce the sensitivity of the solution to the value of  $\delta(s)$  at large  $s$ , as shown in Eq. (6). As already stressed, this possibility is presently rather remote. In practice, the integral entering  $O(s)$  has to be restricted to the elastic region, unless some further assumptions on the amplitudes of the other channels are made. These issues entail some uncertainties which have been partially discussed in Ref. [11].

Under these assumptions, we now envisage a possible implementation of this approach in lattice simulations. It has been shown that, even in the presence of chiral symmetry breaking and of the Maiani-Testa theorem [22], lattice calculations can obtain the physical matrix elements at threshold, corresponding to  $s = 4m_\pi^2$  [23]. This information, once available, combined with the constraint  $A(s = m_\pi^2) = 0$  and with the ansatz that the amplitude grows linearly in  $s$ , is sufficient to determine the physical result for the real kaon mass (see Eq. (8)) in the case of  $(8_L, 1_R)$  operators.

Concerning  $\Delta I = 3/2$  matrix elements of  $(8_L, 8_R)$  operators, one may argue that one condition is not enough to fully determine the amplitude. We stress, however, that in this case the extrapolation between the  $\pi\pi$  threshold and the physical kaon mass is expected to be smoother, resulting in a correction factor which can be approximated by unity with high accuracy.

The evaluation of the amplitude at  $s > 4m_\pi^2$  is particularly difficult with lattice simulations since these are performed in the Euclidean space-time. Proposals, however, exist for overcoming this problem [24, 25]. We finally stress that the region  $s < 4m_\pi^2$  is not accessible to lattice calculations, as well as to any other approach, if the particles ought to be on-shell and the inserted operator carries zero momentum.

## 5 Conclusions

We have critically discussed the issue of final state interactions effects in the evaluation of the ratio  $\varepsilon'/\varepsilon$ . Whereas we cannot exclude possible substantial enhancements of  $\varepsilon'/\varepsilon$  through FSI over lattice and large- $N_C$  estimates, our analysis demonstrates that the present calculation of the dispersive factors  $\mathcal{R}_I$  is subject to considerable uncertainties, which prevent to draw any definite conclusion on the importance of these effects.

We summarize the main point of this paper. While the full amplitude clearly does not depend on the choice of the value of  $s$  at which the conditions on the amplitude are placed, the dispersive correction factors are obviously dependent on this choice. As presently we are not in a position to state precisely which value of  $s$  the existing non-perturbative calculations correspond to, it is not possible to determine uniquely the corresponding factors  $\mathcal{R}_I$  to be incorporated in each of those methods.

On the other hand, we have shown that the Muskhelishvili-Omnès approach could be useful to reduce, or at least to better estimate, the uncertainties on the physical amplitudes, once lattice results on the matrix elements at threshold will be available.

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